REVERSE STRESS TESTING FROM A MACROECONOMIC VIEWPOINT: Quantitative Challenges & Solutions for its Practical Implementation

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The Financial Services Authority defines reverse stress tests as *"tests that require a firm to assess scenarios and circumstances that would render its business model unviable, thereby identifying potential business vulnerabilities. Reverse stress-testing starts from an outcome of business failure and identifies circumstances where this might occur. This is different to general stress and scenario testing which tests for outcomes arising from changes in circumstances."*

At its core, reverse stress testing (RST) proposes to "invert" the standard process; starting now from an "outcome" (business failure) with the aim of finding potential states-of-nature (scenarios) consistent with such outcome. When implementing RST in a quantitative way, **multiplicity** emerges as a challenge. The same outcome (for example, high expected losses) could materialize under multiple combinations of risk factors — such as probability of default (PD), exposure at default (EAD), and loss given default (LGD) — and under alternative macroeconomic scenarios.

Multiplicity should definitely pose a concern, as the reverse engineering exercise can end-up identifying only a subset of scenarios that are consistent with the starting assumption. There is risk that such **scenarios that were not found** could be of more relevance — and severity — compared with the ones that were identified by the process.

1 — Multiplicity from a Mathematical Viewpoint

Type-1 Multiplicity: Indeterminacy

Most RST frameworks are faced with the task of matching a large number of risk and macroeconomic variables with a limited set of assumptions; i.e., # variables > # equations. Indeterminacy takes the form of a **continuum** of solutions (scenarios) whose mathematical properties will help the modeler identify avenues to close the extra degrees of freedom. This indeterminacy needs to be dealt with before any further attempt is made to successfully reverse engineer the process. Solutions will require (a) additional ad-hoc assumptions on some parameters (expert-judgment, market-wide assumptions or values in line with regulatory guidelines), and/or (b) additional equations based on empirical findings; e.g., $LGD=f(PD)$. The task is to close the "degree of indeterminacy" to zero and end-up with as many equations as unknowns.

Type-2 Multiplicity: Inverse Mapping

Even after closing the gap between equations and unknowns a new challenge emerges when trying to reverse engineer a process — the inverse of a function may not behave as a function. Consider a stressed value of the risk factors, say x_0 , that is mapped to a vector of outcomes, say $y_0 = f(x_0)$. Mapping y_0 back (i.e., reverse engineering the process) could give us a value of x that is different from x_0 : there may exist another vector x_1 that is consistent with the same outcome $y_0 = f(x_1)$. Specific characteristics of the stress testing process $f: X \rightarrow Y$ will help us ensure that (at least locally) one can "invert" the process and obtain a reliable RST mapping $f^{-1}: Y \rightarrow X$. Applications of results such as "Inverse Function" and "Implicit Function" theorems are applied to understand the shape of the solution-set $\tilde{X} = \{\tilde{x} \in X \subseteq R^n \wedge \tilde{x} \in f^{-1}(y_0)\}$. Under some "regularity" conditions for f, the set $\tilde{X}$ will locally map $X \rightarrow Y$ (stress testing) and $Y \rightarrow X$ (reverse stress testing) in a one-to-one smooth fashion (overcoming type-2 multiplicity).

2 — Multiplicity from a Practical Viewpoint

Handling Type-1 Multiplicity: Factor & Principal Components Analysis

By leveraging on the strong correlation of macroeconomic series, a modeler can concentrate on a smaller set of instruments (factors) that can still replicate most of the variability of the whole sample. The main advantage is the reduction of the "scenario-space" (to avoid indeterminacy). The challenge, however, could be the lack of interpretation for the factors.

We carried a case study to link the top factors to specific macroeconomic series, ensuring that intuition is kept on the nature of the factors while reducing the chances of indeterminacy. Below are the findings for the UK; similar results were found for US and Germany.

Table I: UK factor analysis– Top 21 macroeconomic series

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Factor analysis/correlation                        Number of obs    =        48
    Method: principal factors                      Retained factors =        14
    Rotation: (unrotated)                          Number of params =       203
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Factor	Eigenvalue	Difference	Proportion	Cumulative
Factor1	9.58311	5.68839	0.4995	0.4995
Factor2	3.89472	2.17554	0.2030	0.7025
Factor3	1.71918	0.32750	0.0896	0.7921
Factor4	1.39168	0.42675	0.0725	0.8646
Factor5	0.96493	0.34703	0.0503	0.9149
Factor6	0.61790	0.18001	0.0322	0.9471
Factor7	0.43789	0.15421	0.0228	0.9700
Factor8	0.28368	0.07129	0.0148	0.9847
Factor9	0.21239	0.06647	0.0111	0.9958
Factor10	0.14592	0.08320	0.0076	1.0034
Factor11	0.06272	0.02724	0.0033	1.0067
Factor12	0.03548	0.01399	0.0018	1.0085
Factor13	0.02149	0.01927	0.0011	1.0097
Factor14	0.00222	0.00844	0.0001	1.0098
Factor15	-0.00622	0.00523	-0.0003	1.0095
Factor16	-0.01145	0.00397	-0.0006	1.0089
Factor17	-0.01542	0.00627	-0.0008	1.0081
Factor18	-0.02169	0.00980	-0.0011	1.0069
Factor19	-0.03149	0.01253	-0.0016	1.0053
Factor20	-0.04402	0.01326	-0.0023	1.0030
Factor21	-0.05728	.	-0.0030	1.0000

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LR test: independent vs. saturated: chi2(210) = 1443.70 Prob>chi2 = 0.0000
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The top five UK factors explain over 90% of the variability of the whole sample, with factors 1 and 2 explaining 50% and 20%, respectively. This is encouraging news from a RST angle; almost all of the information embedded in the UK economic cycle can be replicated by these five factors. But what do these factors represent? Figures I to VI show the correlation of the factors to specific economic indicators.

Figure I

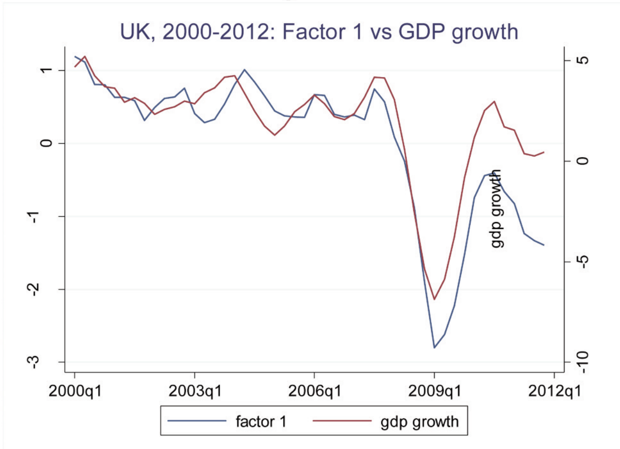


Figure II

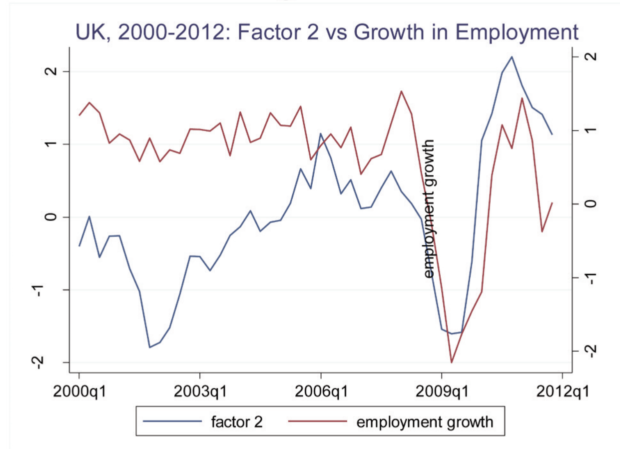


Figure III

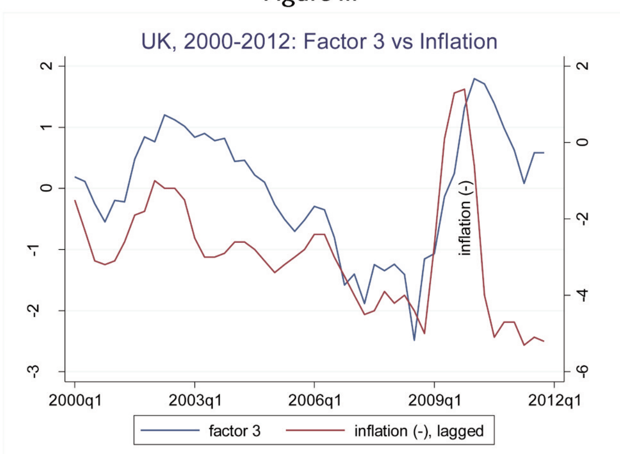


Figure IV

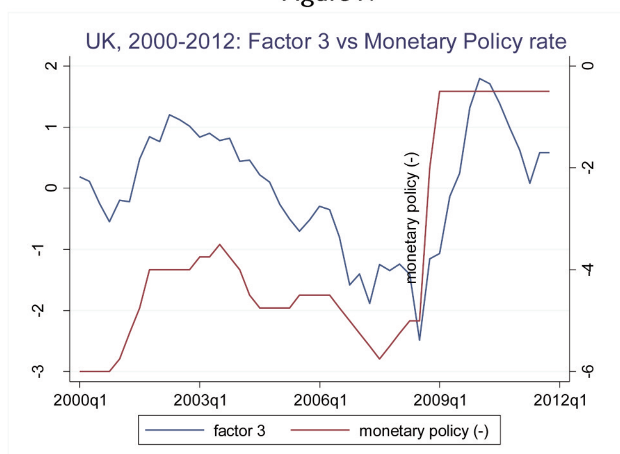


Figure V

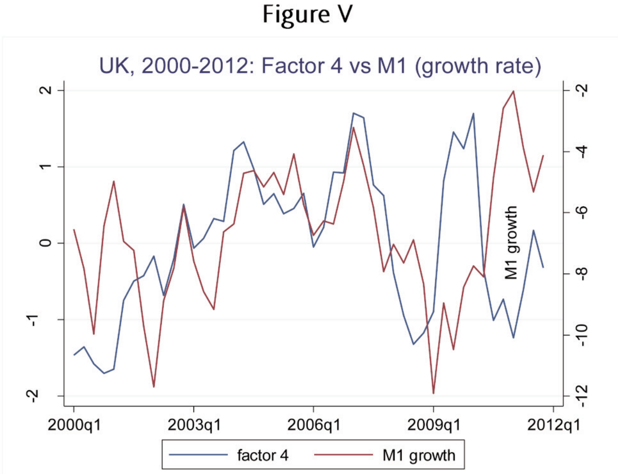
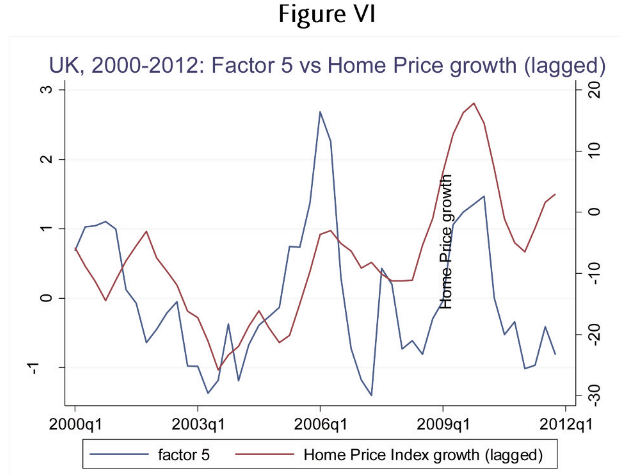


Figure VI



The dimension of the "scenario-space" has been dramatically reduced while keeping an interpretation of the factors: (1) core real business cycle fluctuations, (2) labor market, (3) monetary cycle (inflation and rates), (4) money supply (complementing factor 3), and (5) UK housing market. With this one-to-one match between macro variables and factors, the modeler responsible for RST has a better chance of succeeding, due to the lower number of variables to be matched with outcomes/targets.

2.2 — Handling Type-2 Multiplicity: The Case of Linear Models

Risk modelers usually apply non-linear transformations to risk variables (e.g., logistic or logarithmic mappings) and then model these transformed series in a linear fashion against macro-economic and other risk drivers. Properties of the coefficient matrices of these linear systems will determine whether the process can be "inverted". The simplest 1-dimensional linear model will require a non-zero estimated coefficient. When dealing with higher order systems, the non-zero condition translates to the determinant of a specific matrix.

Consider a linear model: $Y_I = \beta X_I + \varepsilon_I$, where $Y_I \in R^M$ represents a vector of risk variables or targets and $X_I \in R^N$ contains all the model drivers. The (one-step-ahead) forecast takes the form $Y^F_{I+1} = \hat{\beta} X_{I+1}$, with $\hat{\beta}$ representing the estimated parameters.

Suppose now that the RST mandate is to start with an assumption for the outcome, say Y^s_{I+1}, and find consistent values for X_{I+1}. This implies $(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}'Y^s_{I+1} = X_{I+1}$. If $N > M$, we are back into type-1 multiplicity; so $N = M$ is necessary to continue working on the RST process. With an equal number of targets and control variables, the necessary and sufficient condition that needs to be verified is the full rank of $\hat{\beta}'\hat{\beta}$ (or a non-zero determinant). Ensuring that $\hat{\beta}'\hat{\beta}$ is "invertible" will avoid the presence of type-2 multiplicity and the scenarios in X_{I+1} will be uniquely linked to the outcome Y_{I+1}.

With non-linear transformations of the original risk variables that are **strictly monotone** (as it is the case for logarithmic and logistic mappings), the modeler is able to avoid type-2 multiplicity and the RST exercise can be carried forward.

ABOUT THE AUTHORS

Dr. Juan M. Licari is a Senior Director at Moody's Analytics and the head of the Credit Analytics team for Europe, the Middle East, and Africa. Dr. Licari's team provides consulting support to major industry players, builds econometric tools to model credit phenomena, and implements several stress-testing platforms to quantify portfolio risk exposure. His team is an industry leader in developing and implementing credit solutions that explicitly connect credit data to the underlying economic cycle, allowing portfolio managers to plan for alternative macroeconomic scenarios.

Juan is actively involved in communicating the team's research and methodologies to the market. He often speaks at credit events and economic conferences worldwide. Dr. Licari holds a PhD and an MA in economics from the University of Pennsylvania and graduated summa cum laude from the National University of Cordoba in Argentina.

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